THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics

MATH2230 Complex Variables with Applications
2024-2025 Term 2

Homework 7
Due time: 10th March 2025, 23:59

All problems are taken from COMPLEX VARIABLES AND APPLICATIONS, Ninth Edition, by James Ward Brown/Ruel V. Churchill.

P.170: 1, 3, 7, 10
P.177: 2, 4, 5

Practice Problems (Do not turn in)

P.171: 8, 9

Please also find the Homework problems below and email me (xiaolilin@cuhk.edu.hk) if there is any typo.

1. (**P.170, Q1**) Let C denote the positively oriented boundary of the square whose sides lie along the lines $x = \pm 2$ and $y = \pm 2$. Evaluate each of these integrals

   (a)
   $$\int_C \frac{e^{-z}dz}{z-(\pi i/2)}$$
   (b)
   $$\int_C \frac{\cos z}{z(z^2+8)}dz$$
   (c)
   $$\int_C \frac{zdz}{2z+1}$$

(d)

$$\int_C \frac{\cosh z}{z^4} dz$$

(e)

$$\int_C \frac{\tan(z/2)}{(z-x_0)^2} \quad (-2 < x_0 < 2)$$

2. (**P.170, Q3**) Let C be the circle $|z| = 3$, described in the positive sense. Show that if

$$g(z) = \int_C \frac{2s^2 - s - 2}{s - z} ds \quad (|z| \neq 3)$$

then $g(2) = 8\pi i$. What is the value of $g(z)$ when $|z| > 3$?

3. (**P.170, Q7**) Let C be the unit circle $z = e^{i\theta}(-\pi \leq \theta \leq \pi)$. First show that for any real constant a,

$$\int_C \frac{e^{az}}{z} dz = 2\pi i$$

Then write this integral in terms of θ to derive the integration formula

$$\int_0^\pi e^{a\cos\theta} \cos(a\sin\theta) d\theta$$

4. (**P.170, Q10**) Let f be an entire function such that $|f(z)| \leq A|z|$ for all z, where A is a fixed positive number. Show that $f(z) = a_1 z$, where a_1 is a complex constant.

5. (**P.177, Q2**) Let a function f be continuous on a closed bounded region R, and let it be analytic and not constant throughout the interior of R. Assuming that $f(z) \neq 0$ anywhere in R, prove that $|f(z)|$ has a minimum value m in R which occurs on the boundary of R and never in the interior. Do this by applying the corresponding result for maximum values to the function $g(z) = 1/f(z)$.

6. (**P.177, Q4**) Let R region $0 \leq x \leq \pi, 0 \leq y \leq 1$. Show that the modulus of the entire function $f(z) = \sin z$ has a maximum value in R at the boundary point $z = (\pi/2) + i$.

7. (**P.177, Q5**)

   Let $f(z) = u(x,y) + iv(x,y)$ be a function that is continuous on a closed bounded region R and analytic and not constant throughout the interior of R. Prove that the component function $u(x,y)$ has a minimum value in R which occurs on the boundary of R and never in the interior